

Effects of surface vibrations on quadrupole response of nuclei

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Abstract

The effect of quadrupole-type surface vibrations on the quadrupole response function of heavy nuclei is studied by using a model based on the solution of the linearized Vlasov equation with moving-surface boundary conditions. By using a separable approximation for the residual interaction, an analytical expression is obtained for the moving-surface response function. Comparison of the fixed- and moving-surface strength functions shows that surface vibrations are essential in order to achieve a unified description of the two characteristic features of the quadrupole response: the giant resonance and the low-lying states. Calculations performed by setting the surface tension equal to zero shows that the low-lying strength is strongly affected by the surface tension.

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I. INTRODUCTION

There are two systematic features in the quadrupole excitation spectrum of heavy and medium-heavy nuclei: the giant quadrupole resonance at an energy $\hbar\omega \approx 63A^{-\frac{1}{3}}\text{MeV}$ and the lower-energy states that have often been interpreted as surface vibrations. However the exact nature of these states is still under debate since they are strongly affected by shell effects and this may be taken as an indication that they are not pure surface modes (see e. g. [1], p.14).

There is a vast literature on the quadrupole response of nuclei, here we refer only to the nice pedagogical introduction in [2] and to a recent paper [3] where the effect of coupling between the motion of individual nucleons and surface oscillations has been studied in a model that allows also for non-linear effects and for collisions between nucleons. The scope of our present work is more restricted than that of [3], we aim at studying only the effects of coupling between the motion of individual nucleons and surface oscillations of the quadrupole type. Our approach is semiclassical and is based on the solution of the linearized Vlasov kinetic equation. Solutions of this equation for finite systems have been obtained by using different boundary conditions (fixed- and moving-surface [4,5], see also [6]), while the fixed-surface solution can give a reasonable picture of the giant quadrupole resonance, it does

not satisfactorily describe also the low-lying states, however a unified description can be achieved within this model if the moving-surface boundary conditions introduced in [5] are employed. Physically, this means that we are including in our model a coupling between the motion of individual nucleons and the surface vibrations. In our approach this coupling does not involve surface vibrations of different multipolarity. Thus, the present model does not include effects that are not taken into account also in the random-phase approximation.

II. FORMALISM

A. Fixed-surface solution

Within the fixed-surface theory, and assuming a simplified residual interaction of separable form,

$$v(r_1, r_2) = \kappa_2 r_1^2 r_2^2, \quad (2.1)$$

the quadrupole response function of a spherical nucleus described as a system of A interacting nucleons contained within a cavity of radius $R = 1.2A^{\frac{1}{3}}$ fm is given by [7]

$$\mathcal{R}_{22}(s) = \frac{\mathcal{R}_{22}^0(s)}{1 - \kappa_2 \mathcal{R}_{22}^0(s)}, \quad (2.2)$$

where $s = \frac{\omega R}{v_F}$ is a convenient dimensionless variable (v_F is the Fermi velocity). The zero-order response function $\mathcal{R}_{22}^0(s)$ is analogous to the single-particle response function of the quantum theory and is given explicitly by [8]

$$\mathcal{R}_{22}^0(s) = \frac{9A}{8\pi} \frac{1}{\epsilon_F} \sum_{n=-\infty}^{+\infty} \sum_{N=0, \pm 2} C_{2N}^2 \int_0^1 dx x^2 s_{nN}(x) \frac{(Q_{nN}^2(x))^2}{s + i\varepsilon - s_{nN}(x)}. \quad (2.3)$$

Here ϵ_F is the Fermi energy, the coefficients C_{2N}^2 are $C_{20}^2 = \frac{1}{4}$ and $C_{2\pm 2}^2 = \frac{3}{8}$. The functions $s_{nN}(x)$ are defined as

$$s_{nN}(x) = \frac{n\pi + N \arcsin(x)}{x} \quad (2.4)$$

and the quantity ε is a vanishingly small parameter that determines the integration path at poles. The Fourier coefficients $Q_{nN}^2(x)$ are the classical limit of the quantal radial matrix elements and are given by:

$$Q_{nN}^2(x) = (-)^n R^2 \frac{2}{s_{nN}^2(x)} \left(1 + N \frac{\sqrt{1-x^2}}{s_{nN}(x)} \right) \quad \text{for } (n, N) \neq (0, 0) \quad (2.5)$$

and

$$Q_{00}^2(x) = R^2 \left(1 - \frac{2}{3} x^2 \right). \quad (2.6)$$

The response function (2.3) involves an infinite sum over n , however in practice it is sufficient to include only a few terms around $n = 0$ in order to fulfill the energy-weighted sum rule with good accuracy. The form (2.6) of the coefficient Q_{00}^2 implies that the term $(n, N) = (0, 0)$ does not contribute to the strength function, hence this term can be omitted from the sum in Eq. (2.3).

B. Moving-surface solution

Within the moving-surface theory of [5], the collective response function (2.2) is replaced by

$$\tilde{\mathcal{R}}_{22}(s) = \mathcal{R}_{22}(s) + \mathcal{S}_{22}(s), \quad (2.7)$$

with $\mathcal{R}_{22}(s)$ still given by Eq. (2.2), while $\mathcal{S}_{22}(s)$ represents the moving-surface contribution. With the simple interaction (2.1) the function $\mathcal{S}_{22}(s)$ can be evaluated explicitly as (see Appendix)

$$\mathcal{S}_{22}(s) = -\frac{R^6}{1 - \kappa_2 \mathcal{R}_{22}^0(s)} \frac{[\chi_2^0(s) + \kappa_2 \varrho_0 R^2 \mathcal{R}_{22}^0(s)]^2}{[C_2 - \chi_2(s)][1 - \kappa_2 \mathcal{R}_{22}^0(s)] + \kappa_2 R^6 [\chi_2^0(s) + \varrho_0 R^2]^2}, \quad (2.8)$$

with $C_2 = 4\sigma R^2$ ($\sigma \approx 1\text{MeV fm}^{-2}$ is the surface tension parameter obtained from the mass formula) and $\varrho_0 = A/\frac{4\pi}{3}R^3$ the equilibrium density.

The functions $\chi_2^0(s)$ and $\chi_2(s)$ are defined as in Refs. [8] and [9] and are given by

$$\chi_2^0(s) = \frac{9A}{4\pi} \frac{1}{R^3} \sum_{n=-\infty}^{+\infty} \sum_{N=0, \pm 2} C_{2N}^2 \int_0^1 dx x^2 s_{nN}(x) \frac{(-)^n Q_{nN}^2(x)}{s + i\varepsilon - s_{nN}(x)}, \quad (2.9)$$

and

$$\begin{aligned} \chi_2(s) = & -\frac{9A}{2\pi} \epsilon_F (s + i\varepsilon) \left\{ \frac{1}{4} \int_0^1 dx x^3 \cot[(s + i\varepsilon)x] + \frac{3}{8} \int_0^1 dx x^3 \left(\cot[(s + i\varepsilon)x - 2 \arcsin(x)] \right. \right. \\ & \left. \left. + \cot[(s + i\varepsilon)x + 2 \arcsin(x)] \right) \right\}. \end{aligned} \quad (2.10)$$

Equation (2.8) is the main result of the present paper, its explicit derivation is lengthy but straightforward, the main steps are outlined in the Appendix. Together with Eq. (2.7), Eq. (2.8) gives a unified expression for the quadrupole response function, including both the high-energy giant resonance and the low-energy excitations. By comparing the two response functions (2.2) and (2.7) we can appreciate the effect of the additional surface degree of freedom introduced in [5] and in particular the effect of coupling the motion of nucleons with surface vibrations of quadrupole type.

III. RESULTS

In Fig.1 we display the strength function ($E = \hbar\omega$)

$$S(E) = -\frac{1}{\pi} \text{Im } \mathcal{R}(E), \quad (3.1)$$

obtained for $A = 208$ using different approximations. The dotted curve is obtained from the zero-order response function (2.3) and it is similar to the quantum response evaluated in the Hartree-Fock approximation. The dashed curve is obtained from the collective fixed-surface response function (2.2). Comparison with the dotted curve clearly shows the effects of collectivity: the main strength at about 16 MeV is shifted to lower excitation energy and

the giant quadrupole peak becomes narrower. The strength of the interaction (2.1), chosen in order to reproduce the experimental value of the giant quadrupole resonance energy in ^{208}Pb , is $\kappa_2 = -1.10^{-3}\text{MeV fm}^{-4}$. this value is close to that suggested by the Bohr-Mottelson prescription ([10], p. 509)

$$\kappa_2 = -\frac{4\pi}{3} \frac{m\omega_0^2}{AR^2} \approx -0.510^{-3}\text{MeV fm}^{-4}, \quad (3.2)$$

(with $\omega_0 = 41A^{-\frac{1}{3}}\text{MeV}$).

We notice that the width of the giant quadrupole resonance is underestimated by the fixed-surface model, this is a well known limit of all mean-field calculations that include only Landau damping. Moreover, there is no sign of a low-energy peak in the fixed-surface response function. The solid curve instead shows the moving-surface response given by Eqs. (2.7) and (2.8). Now a broad bump appears in the low-energy part of the response and a narrower peak is situated at the giant resonance energy, thus the moving-surface solution of the Vlasov equation introduced in Ref. [5] accounts for both quadrupole modes, although only qualitatively. Of course the details of the low-energy excitations are determined by quantum effects, nonetheless the present semiclassical approach does reproduce the average behaviour of this systematic feature of the quadrupole response.

Another remarkable feature of the moving-surface response function is that now the giant quadrupole peak is narrower than for the fixed-surface solution. This is somewhat surprising since we could have expected that introducing a further degree of freedom would result in a smearing of the peak, however our result for the giant resonance region is very similar to that of the recent random-phase approximation (RPA) calculations of [11] (cf. Fig.5 of [11]). Our model does include Landau damping that, however, turns out to be very small in this case, clearly some additional mechanism is required in order to increase the width of the giant resonance. Two such mechanisms have been considered in Refs. [12,3], they are the coupling to surface vibrations of different multipolarity and the effect of collisions between nucleons. It would be interesting to include such effects in the present semiclassical theory, however this will be left for future work.

All the strength functions shown in Fig.1 should satisfy the following energy-weighted sum rule (EWSR) (see e.g. [10], p. 401):

$$\int_0^\infty dE E S(E) = \frac{3}{4\pi} \frac{\hbar^2}{m} AR^2. \quad (3.3)$$

We have numerically checked that, when integrated up to $E = 30\text{MeV}$, the response function shown by the solid curve exhausts about 98% of this sum rule. The fraction of EWSR exhausted by the dashed and dotted strength functions in the same interval is only 80%, showing that in these cases there is some more strength at higher energy.

Another interesting moment of the strength function is the inverse energy-weighted sum rule:

$$m_{-1} = \int_0^\infty dE \frac{S(E)}{E}. \quad (3.4)$$

From the three strength functions shown in Fig.1, we have three different inverse moments :

$$m_{-1}^0 = -\frac{1}{2} \lim_{s \rightarrow 0} \mathcal{R}_{22}^0(s) = \frac{139}{140} \frac{1}{16\pi} \frac{AR^4}{\epsilon_F} \quad (3.5)$$

(when evaluating $\lim_{s \rightarrow 0} \mathcal{R}_{22}^0(s)$ the term $(n, N) = (0, 0)$ must be omitted from the sum in Eq. (2.3)),

$$m_{-1} = m_{-1}^0 \frac{1}{1 + 2\kappa_2 m_{-1}^0} \quad (3.6)$$

and

$$\tilde{m}_{-1} = m_{-1} \left\{ 1 + 12 \frac{140}{139} \frac{\left[\frac{17}{20} + 2\kappa_2 m_{-1}^0 \right]^2}{\left[1 + 2\kappa_2 m_{-1}^0 \right] \left[1 + \frac{8}{3} \frac{4\pi\sigma R^2}{A\epsilon_F} + \frac{27}{50} \frac{140}{139} \kappa_2 m_{-1}^0 \right]} \right\}. \quad (3.7)$$

The inverse moment of the zero-order strength function (dotted curve in Fig.1) m_{-1}^0 exhausts about 97% of the sum rule (3.5) in the range from 0 to 30 MeV. The collective fixed-surface response (dashed curve in Fig.1) instead exhausts almost 99% of the sum rule (3.6) in the same energy range while the moving-surface strength function exhausts almost 100% of the sum rule (3.7), always in the same energy interval.

It can be seen from Fig.1 that, while the fixed-surface response has only one collective pole, the moving-surface quadrupole response function displays a two-pole structure. In order to get more information about the nature of the low-energy peak, we have performed calculations by putting the surface tension parameter σ equal to zero. The result is shown in Fig.2, where the dotted curve corresponds to $\sigma = 0$. As expected, the giant resonance peak is practically unaffected by the surface tension, while the low-energy peak is affected quite substantially. The surface tension increases the frequency of the low-energy peak, which, however, is present at a non-vanishing frequency also in the absence of surface tension. In the opposite limit, if we let $\sigma \rightarrow \infty$, the fixed-surface response is obtained.

We have performed calculations of the quadrupole response functions also for other values of A corresponding to medium-heavy spherical nuclei and the results are qualitatively similar to the $A = 208$ case, so we do not report them here.

A calculation of the isoscalar quadrupole response similar to the present one has been made in Ref. [14], in that case however a rather special external field has been assumed. The external force studied in [14] is a pressure that acts only on the surface of the nucleus, perhaps this explains why in that case very little strength was found in the region of the giant quadrupole resonance.

IV. SUMMARY AND CONCLUSIONS

We have obtained an analytical expression for the isoscalar quadrupole response function of nuclei that describes qualitatively the two main systematic features of the excitation spectrum: the giant resonance and the low-energy states. Our approach is semiclassical and is based on the solution of the linearized Vlasov kinetic equation with appropriate boundary conditions (moving surface). Comparison of our result (full curve) with the quantum response function of Ref. [11] shows that quantum effects modify substantially the low-energy region (a discrete state is obtained in [11] instead of our broad bump), while the giant resonance peak is practically the same in the quantum and semiclassical approach.

APPENDIX: MOVING-SURFACE RESPONSE

In our semiclassical approach the nucleus is described by means of the phase-space density $f(\mathbf{r}, \mathbf{p}, t)$. At equilibrium the density is $f_0(\mathbf{r}, \mathbf{p}) = F(h_0(\mathbf{r}, \mathbf{p}))$, where h_0 is the equilibrium mean-field hamiltonian. A weak external driving field $V_{ext}(\mathbf{r}, t) = \beta(t)Q(\mathbf{r})$ induces small fluctuations of the equilibrium distribution f_0 and of the mean field. Since the two fluctuations are related, we have a typical self-consistency problem. For spherical system the problem can be solved by means of the partial-wave expansion [4,6]

$$\delta f(\mathbf{r}, \mathbf{p}, \omega) = \sum_{LMN} \left[\delta \tilde{f}_{MN}^{L+}(\epsilon, \lambda, r, \omega) + \delta \tilde{f}_{MN}^{L-}(\epsilon, \lambda, r, \omega) \right] \left(\mathcal{D}_{MN}^L(\alpha, \beta, \gamma) \right)^* Y_{LN}\left(\frac{\pi}{2}, \frac{\pi}{2}\right), \quad (\text{A1})$$

where $\mathcal{D}_{MN}^L(\alpha, \beta, \gamma)$ are the Wigner rotation matrices, $\epsilon = h_0$ is the particle energy and λ its angular momentum.

In the approach of [5] the functions $\delta \tilde{f}_{MN}^{L\pm}$ satisfy the integral equation (see Ref. [6] for details)

$$\begin{aligned} \delta \tilde{f}_{MN}^{L\pm}(\epsilon, \lambda, r, \omega) = & F'(\epsilon) \frac{e^{\pm i[\omega\tau(r) - N\gamma(r)]}}{\sin[\omega\tau(R) - N\gamma(R)]} \omega p_r(R) \delta R_{LM}(\omega) \\ & + \left\{ \int_{r_1}^r dr' \tilde{B}_{MN}^{L\pm}(\epsilon, \lambda, r') e^{\mp i[\omega\tau(r') - N\gamma(r')]} + \tilde{C}_{MN}^L(\epsilon, \lambda, \omega) \right\} e^{\pm i[\omega\tau(r) - N\gamma(r)]}, \end{aligned} \quad (\text{A2})$$

with

$$\tilde{B}_{MN}^{L\pm}(\epsilon, \lambda, r, \omega) = F'(\epsilon) \left(\frac{\partial}{\partial r} \pm \frac{iN}{v_r(\epsilon, \lambda, r)} \frac{\lambda}{mr^2} \right) \left[\beta(\omega) Q_{LM}(r) + \delta \tilde{V}_{LM}(r, \omega) \right], \quad (\text{A3})$$

$$\begin{aligned} \delta \tilde{V}_{LM}(r, \omega) = & \frac{8\pi^2}{2L+1} \sum_{N=-L}^L \left| Y_{LN}\left(\frac{\pi}{2}, \frac{\pi}{2}\right) \right|^2 \\ & \times \int d\epsilon \int d\lambda \lambda \int \frac{dr'}{v_r(r')} v_L(r, r') [\delta \tilde{f}_{MN}^{L+}(\epsilon, \lambda, r', \omega) + \delta \tilde{f}_{MN}^{L-}(\epsilon, \lambda, r', \omega)] \end{aligned} \quad (\text{A4})$$

and

$$\begin{aligned} \tilde{C}_{MN}^L(\epsilon, \lambda, \omega) = & \left\{ e^{i2[\omega\tau(R) - N\gamma(R)]} \int_{r_1}^R dr \tilde{B}_{MN}^{L+}(\epsilon, \lambda, r) e^{-i[\omega\tau(r) - N\gamma(r)]} \right. \\ & \left. - \int_{r_1}^R dr \tilde{B}_{MN}^{L-}(\epsilon, \lambda, r) e^{i[\omega\tau(r) - N\gamma(r)]} \right\} \left\{ 1 - e^{i2[\omega\tau(R) - N\gamma(R)]} \right\}^{-1}, \end{aligned}$$

which is equivalent to the linearized Vlasov equation with the moving-surface boundary conditions

$$\delta \tilde{f}_{MN}^{L+}(R) - \delta \tilde{f}_{MN}^{L-}(R) = 2F'(\epsilon) i\omega p_r(R) \delta R_{LM}(\omega). \quad (\text{A5})$$

The surface fluctuations $\delta R_{LM}(\omega)$ are related to the functions $\delta \tilde{f}_{MN}^{L\pm}$ by

$$\delta R_{LM}(\omega) = \frac{8\pi^2}{2L+1} \frac{R^2}{C_L} \sum_{N=-L}^L \left| Y_{LN}\left(\frac{\pi}{2}, \frac{\pi}{2}\right) \right|^2 \quad (\text{A6})$$

$$\int d\epsilon \int d\lambda \lambda p_r(R) \left[\delta \tilde{f}_{MN}^{L+}(\epsilon, \lambda, R, \omega) + \delta \tilde{f}_{MN}^{L-}(\epsilon, \lambda, R, \omega) - 2F'(\epsilon) \delta \tilde{V}_{LM}(R, \omega) \right].$$

For a separable interaction of the multipole-multipole type, like (2.1), the integral equation (A2) can be reduced to an algebraic equation that, in the particular case $L = 2$ and $V_{ext} = \beta(t)r^2 Y_{2M}(\hat{\mathbf{r}})$, gives Eqs.(2.7) and (2.8) for the quadrupole response function.

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FIGURES

FIG. 1. Quadrupole strength function for a hypothetical nucleus of $A = 208$ nucleons. The dotted curve shows the zero-order (static mean field) approximation, the dashed curve instead shows the collective response evaluated in the fixed-surface approximation. The full curve gives the moving-surface response.

FIG. 2. The full curve is the same as in Fig.1, the dotted curve has been obtained in the moving-surface approach for vanishing surface tension.



